Hydrological Sciences Journal
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/thsj20

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Daniel Barrera [a] & Sergio Masuelli [b]
[a] CONICET/Departamento de Ingeniería Agrícola y Uso de la Tierra, Facultad de Agronomía, Universidad de Buenos Aires, Av. San Martín 4453, Ciudad de Buenos Aires, 1417, Argentina
[b] Instituto de Altos Estudios Espaciales "Mario Gulich", CETT/CONAE, Ruta C45 - Km 8, Falda del Carmen, 5187, Argentina
Published online: 12 Jul 2011.

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Daniel Barrera[1] & Sergio Masuelli[2]

[1]*CONICET/Departamento de Ingeniería Agrícola y Uso de la Tierra, Facultad de Agronomía, Universidad de Buenos Aires, Av. San Martín 4453, Ciudad de Buenos Aires, 1417 Argentina*
dbarrera@agro.uba.ar

[2]*Instituto de Altos Estudios Espaciales "Mario Gulich", CETT/CONAE, Ruta C45 - Km 8, Falda del Carmen, 5187 Argentina*
sergio.masuelli@conae.gov.ar

Received 27 March 2007; accepted 2 February 2011; open for discussion until 1 January 2012

Citation Barrera, D. & Masuelli, S. (2011) An extension of the Green-Ampt model to decreasing flooding depth conditions, with efficient dimensionless parametric solution. *Hydrol. Sci. J.* **56**(5), 824–833.

Abstract The theoretical formulation of the Green-Ampt infiltration model has been extended to conditions of decreasing flooding depth in an isolated system. By defining dimensionless variables of flooding depth s and time τ, an implicit dimensionless equation $\tau(s)$ was obtained, which contains a single fundamental dimensionless parameter γ controlling the process, named "infiltration delay parameter". The characteristics and functional behaviour of γ were analysed, and its physical meaning discussed. A parametric expression $s(\tau)$ has been obtained, which uses a unique descriptive parameter a, which in turn depends only on γ and on four generic coefficients valid for a wide range of soil properties and conditions occurring in nature. By means of numerical simulations using different values of soil parameters and initial flooding depths, it was proved that the proposed parametric function generates similar infiltration rates and cumulative storages to those that are obtained starting from Darcy's equation in the extended Green-Ampt scheme.

Key words infiltration; Green-Ampt model; Darcy's equation; dimensionless variables; parametric solution

Extension du modèle de Green-Ampt aux conditions de hauteur de submersion décroissante, avec solution paramétrique adimensionnelle efficace

Résumé La formulation théorique du modèle d'infiltration de Green-Ampt a été étendue à des conditions de hauteur de submersion décroissante au sein d'un système isolé. En définissant des variables adimensionnelles de hauteur, s et de temps τ de submersion, une équation adimensionnelle implicite $\tau(s)$ a été obtenue, qui comporte un paramètre adimensionnel fondamental simple γ de contrôle du processus, appelé "paramètre de retard de l'infiltration". Les caractéristiques et le comportement fonctionnel de γ ont été analysés et sa signification physique discutée. Une expression paramétrique $s(\tau)$ a été obtenue, qui s'appuie sur un paramètre descriptif unique a, qui dépend à son tour seulement de γ et de quatre coefficients génériques valides pour une large gamme de propriétés et de conditions pédologiques naturelles. Au moyen de simulations numériques basées sur différentes valeurs des paramètres pédologiques et de la hauteur de submersion initiale, il a été prouvé que la fonction paramétrique proposée génère des taux d'infiltration et des stockages cumulés semblables à ceux que l'on obtient avec l'équation de Darcy dans le schéma de Green-Ampt étendu.

Mots clefs infiltration; modèle de Green-Ampt; équation de Darcy; variables adimensionnelles; solution paramétrique

INTRODUCTION

Green and Ampt (1911) proposed a simplified model of the infiltration process that contains realistic physics and, at the same time, enables one to obtain an analytical solution, starting from Darcy's and the continuity equations. They provided the first physically-based equation which describes the infiltration process of water into the soil. They introduced the concept of "sharp wetting front" as a horizontal surface that goes down, leaving behind a saturated soil fringe, while, below the front, the soil remains at its initial water content. The sharp wetting front is a flat and abrupt frontier with a

ISSN 0262-6667 print/ISSN 2150-3435 online

doi: 10.1080/02626667.2011.585137
http://www.informaworld.com

discontinuity in the vertical profiles of water content and of capillary suction. This last variable remains constant at the wetting front independently of its position and of the elapsed time. Later, Philip (1954) gave firm physical arguments to this model.

On another line of theoretical research, Richards (1931) developed an equation for the simplest case of unsaturated flow that expresses the water movement in transient conditions through an unsaturated porous medium, starting from Darcy's and the continuity equations. Richards obtained a second-order partial differential equation that does not have analytical solutions unless some simplifications are made. Several authors developed infiltration equations starting from Richard's work. Philip (1957a) developed a theory of infiltration. Then, making simple assumptions for the hydraulic conductivity and the diffusivity in unsaturated soil, he obtained an ordinary differential equation, and by means of a numerical method he arrived to an approximate solution: a mathematical expression of the infiltration rate as a function of time for a ponding layer of negligible thickness (Philip 1957b).

The Green-Ampt method gives satisfactory results especially when the soil is initially dry, and in particular for soils with a thick texture that exhibit an abrupt wetting front (Hillel 1971). Green and Ampt also assumed that, in the whole soil profile, the porosity and the saturated hydraulic conductivity are uniform. Morel-Seytoux and Khanji (1974) introduced a viscous correction factor, which is a function of the initial soil water content, with a strong variation near saturation. By means of this factor the authors explained the existence of a transition zone instead of the above-mentioned abrupt front. The Green-Ampt scheme has been the object of considerable development in the fields of soil physics and hydrology due to its simplicity and satisfactory versatility for a wide variety of infiltration problems in which the ponding depth can be considered as constant. The implicit equations for the infiltration rate, f, and their cumulative value, F, obtained after integration of Darcy's equation must be solved numerically by means of an iterative method, because the required functions $f(t)$ and $F(t)$ are implicit in the form of transcendental equations. Salvucci and Entekhabi (1994) obtained explicit parametric expressions for these variables by means of further development as time series. With a truncation in the fourth term of the time series obtained for $f(\tau)$, where τ is a dimensionless time variable, they reported satisfactory adjustments for any lapse of time, with errors smaller than 2%.

In the case of flatlands with very small slopes, runoff is very weak and often the rainwater accumulates at the soil surface, flooding it after the rain has ceased, or at least ponding the depressions. In this context, only vertical infiltration is relevant for short time scales; other transfers, such as runoff, subsurface flow and evapotranspiration, become less significant. In this case, the flooding layer decreases its depth with time as the water infiltrates, and therefore the classical Green-Ampt model does not apply. In one case, Philip (1992) treated the case of falling-head ponded infiltration for an isolated system, and arrived at a transcendental equation where the wetting front depth is an implicit function of time. He also obtained an analytical expression for the time it takes the pond to empty. More recently, Warrick *et al.* (2005) applied the classical Green-Ampt model to compute cumulative infiltration under variable ponding depths by considering constant depths for very short intervals and varying them in successive intervals. They found the results "reasonably robust" when comparing them against field-measured values from two irrigation events.

The general objective of this work is to obtain explicit expressions for the decreasing depth of the flooding layer and for the infiltration rate for Darcy's equation in the Green-Ampt "sharp wetting front" model, assuming that a prescribed amount—and depth—of water is initially flooding the terrain and that the only active process is infiltration.

PHYSICAL BASIS AND THEORETICAL FORMULATION

The Green-Ampt scheme for falling-head infiltration

Figure 1 depicts the outline of the Green-Ampt scheme for infiltration for any time t after its beginning, in an isolated system with mass conservation

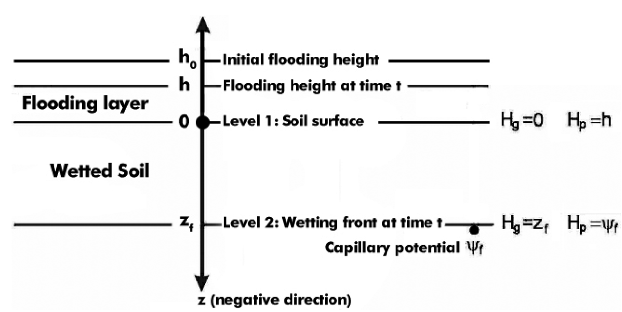

Fig. 1 Green-Ampt model of falling head ponded infiltration and acting gravity and pressure potential heads.

and transfer between the ponded water and the wetted zone.

The variable, $\Delta\theta$, which is the difference between initial and saturated soil water content, expresses the relative volume of pores to be suddenly filled by water during the passage of the sharp wetting front, except for a very small fraction corresponding to the volume of air bubbles trapped in the soil pores during its wetting. The effective saturated conductivity K_e, also called "hydraulic conductivity to re-saturation", is a fraction of the saturated conductivity, K, because of the presence of bubbles which retard the movement of water. According to Vieux (2004), the value of $K_e = 1/2K$ is usually taken, whereas Morel-Seytoux and Khanji (1974) proposed closer values of K and K_e, with a ratio K/K_e that varies between 1.7 and 1.

It is assumed, as in the Green-Ampt scheme, that the soil is initially unsaturated, with the following characteristics:

– The soil matrix is incompressible and the soil texture is homogeneous, which implies that the porosity is constant in time and uniform in the profile.
– The initial soil water content, θ_i, is uniform in the whole profile. Additionally it is assumed that water migration by vapour diffusion does not occur in the soil. Therefore, the saturation deficit, $\Delta\theta$ (or available pore space in the soil) is constant and uniform.
– The capillary potential at the wetting front, ψ_f, is uniform in the vertical profile and constant in time.
– The effective hydraulic conductivity of the wet soil, K_e, is uniform in the vertical profile and constant in time.
– At the beginning of the infiltration, a flooding water layer of height h_0 is present at the surface, with a contiguous layer of wetted soil of infinitesimal thickness (in which Darcy's equation is valid) followed by a deep layer of unsaturated soil. The outline also admits the pre-existence of an already wetted soil layer of given depth contiguous to the surface, as if the process had begun before the instant considered as initial.

Mathematical description of the falling-head infiltration process based on the Green-Ampt model

We start from Darcy's equation in a similar form to that in the Green-Ampt infiltration model—as presented in Chow *et al.* (1994)—considering the potentials of pressure, H_p (hydrostatic pressure and capillarity) and gravity, H_g expressed as units of energy per unit weight, or potential head (in units of length). We call L the soil layer thickness which becomes saturated behind the wetting front; thus, L increases downward, i.e. $L = -z_f$. Also, we denote ψ the suction head at the wetting front, or the capillary potential ψ_f expressed as positive.

Thus, the difference in the potential head at any time, when comparing level 1 (with $z = 0$ at the soil surface) with level 2 ($z = z_f$ at the wetting front), is:

$$\Delta H = \left(H_g + H_p\right)_1 - \left(H_g + H_p\right)_2 = h + L + \psi \quad (1)$$

and $\Delta z = z_1 - z_2 = 0 + L$. Therefore, Darcy's equation for the flux q downward through the wetted soil can be expressed, when replacing the partial derivatives by a difference approximation, as follows:

$$q = -K\left(\frac{\partial H}{\partial z}\right) = -K\left(\frac{h + L + \psi}{L}\right) \quad (2)$$

For simplicity, we use K instead of K_e. The left-hand term in equation (2) expresses the uniform (and negative) flow in the wet soil layer, and has the same absolute value and different sign as the infiltration rate f in the Green-Ampt scheme of the sharp wetting front.

In our case, the decreasing rate of flooding depth is equal to f in absolute value and of opposite sign:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -f = q \quad (3)$$

From equations (2) and (3) we have:

$$-f = \frac{\mathrm{d}h}{\mathrm{d}t} = -K\left(\frac{h + L + \psi}{L}\right) \quad (4)$$

We now propose a treatment of equation (4) which, contrary to the original Green-Ampt model, does not require that the flooding or ponding layer keep a constant thickness. We assume that the flooding layer is continuously reducing in thickness as water infiltrates, changing from an initial value h_0 to a smaller one $h(t)$ at time t.

The Green-Ampt development idealizes the infiltration process for a sharp wetting front, and the cumulative infiltration, F, at a given instant, t, is equal to the product $L \cdot \Delta\theta$. In analogy with this scheme, we assume that at time t, the depth of infiltrated water

$(h_0 - h)$ corresponds to an advancement of the wetting front through a soil layer of thickness, L, which increases its water content by $\Delta\theta$:

$$F = h_0 - h = L \cdot \Delta\theta \quad \Rightarrow L = \frac{h_0 - h}{\Delta\theta} \tag{5}$$

This implies that neither lateral flow (runoff and/or sub-surface flow), nor evapotranspiration exist, or that these fluxes are exactly compensated by additional rain. The relationship assumed in equation (5) was proposed earlier by Philip (1992).

We now define the positive variable $\alpha = 1 - \Delta\theta$. By replacing $\Delta\theta$ in equation (5) and combining it with equation (4), one obtains:

$$\frac{dh}{dt} = -K\left(\frac{h_0 - \alpha\, h + (1-\alpha)\,\psi}{h_0 - h}\right) \tag{6}$$

We also define the dimensionless variable, s:

$$s = \frac{h}{h_0} \tag{7}$$

which expresses the flooding depth relative to its initial value. We now define the auxiliary variable:

$$\chi = 1 + \frac{\Delta\theta \cdot \psi}{h_0} \tag{8}$$

By using these two variables in equation (6), we obtain:

$$\frac{ds}{dt} = -\frac{K}{h_0}\left(\frac{\chi - \alpha\, s}{1-s}\right) \tag{9}$$

We now put the time variable in a dimensionless form:

$$\tau = \frac{K \cdot \chi}{h_0} \cdot t = K\left(\frac{1}{h_0} + \frac{\Delta\theta \cdot \psi}{h_0^2}\right) t \tag{10}$$

and define a new variable:

$$\gamma = \frac{\alpha}{\chi} = \frac{1 - \Delta\theta}{\left(1 + \frac{\Delta\theta \cdot \psi}{h_0}\right)} \tag{11}$$

By using equations (10) and (11) in equation (9) we obtain:

$$\frac{ds}{d\tau} = -\left(\frac{1 - \gamma s}{1 - s}\right) \Leftrightarrow d\tau = -ds\left(\frac{1 - s}{1 - \gamma s}\right) \tag{12}$$

In this differential equation, all the information referring to the soil type and its initial conditions are condensed into the parameter γ and the dimensionless variables s and τ.

Integrating both members of (12), yields:

$$\tau = \frac{\gamma - 1}{\gamma^2}\left[\ln\left(\frac{1 - \gamma s}{1 - \gamma}\right)\right] + \frac{1 - s}{\gamma} \tag{13}$$

This is the equation that we were looking for: the Green-Ampt model dimensionless implicit equation for falling head in an isolated system. Similarly to what happens in the classical approach for constant ponding depth, it is not possible to obtain the inverse expression $s(\tau)$ analytically.

Before solving equation (13), it is interesting to analyse some particular cases. First, we can calculate the dimensionless time, τ_0, for which the pond empties, i.e. $s = 0$:

$$\tau_0 = \frac{1 - \gamma}{\gamma^2}\ \ln(1 - \gamma) + \frac{1}{\gamma} \tag{14}$$

The variation of τ_0 as a function of γ is shown in Fig. 2; γ varies from 1, at saturation, to near 0. In nature, values of γ close to zero occur when the soil is very dry, and therefore ψ is much larger than h_0. It is interesting to note that both τ_0 and γ depend only on the initial conditions in the soil and above it, and they are independent of the hydraulic conductivity.

Functional characteristics and physical meaning of the parameter gamma

The parameter gamma is a dimensionless number which allows the falling-head infiltration equation to be expressed for a sharp wetting front in its simplest form, that is, dimensionless in time and

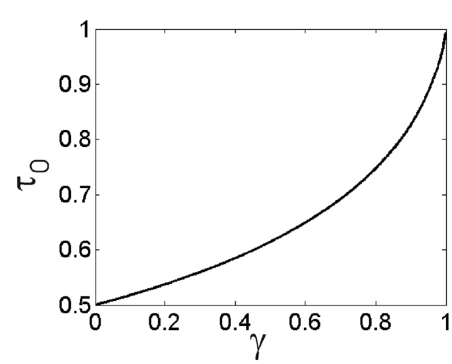

Fig. 2 Dimensionless time at completion of the infiltration process τ_0 *vs* parameter γ.

space, and as a function of a single, physically-based parameter.

It can be seen from equation (11) that γ increases when $\Delta\theta$ and ψ decrease, which in turn implies that the infiltration rate will be slower at all times; also, γ increases with h_0. In both cases, a higher value of γ means that the completion of the infiltration process will take more time—the time needed to empty the flooding water—as shown in equation (14) and Fig. 2.

It is interesting to highlight the functional behaviour of γ varying along with h_0, ψ and $\Delta\theta$, which is shown in Fig. 3. From a functional point of view, it is convenient to treat γ as a function of saturation deficit $\Delta\theta$ and the quotient ψ/h_0 between the front suction to initial hydrostatic heads, because γ depends only on $\Delta\theta$ if this quotient remains constant. In Fig. 3(a), we can see that higher values of $\Delta\theta$ and ψ/h_0 result in lower values of γ. For almost saturated soils, γ takes relatively high values (>0.5) and becomes weakly dependent on ψ/h_0, and inversely for dry soils. In Fig. 3(b) (γ *vs* $\Delta\theta$), the isolines correspond to ψ/h_0; when the soil is initially saturated the variation of γ with $\Delta\theta$ is linear (straight line on top with slope equal to −1), and almost linear for $\psi/h_0 \leq 0.5$. For very high values of ψ/h_0, the fall of γ is very fast for soils near saturation, and very slow for drier soils. Figure 3(c) (γ *vs* ψ/h_0) shows the isolines of $\Delta\theta$. In the range of small values of the quotient ψ/h_0, a very small increase in ψ/h_0 for fixed $\Delta\theta$ causes a rapid decrease in γ, which is faster for drier soils. For $(\psi/h_0) > 20$, the parameter γ is also higher for soils with higher water contents, but it decreases slowly when the quotient between potentials increases. Figure 3(d) shows the isolines of suction head ψ for very dry soils at a fixed saturation deficit ($\Delta\theta = 0.42$), the axes being γ and h_0. The three isolines describe the function $\gamma(h_0)$ for typical suction values of loamy sand ($\psi = 0.05$ m), silt loam ($\psi = 0.15$ m) and clay ($\psi = 0.45$ m). The increase

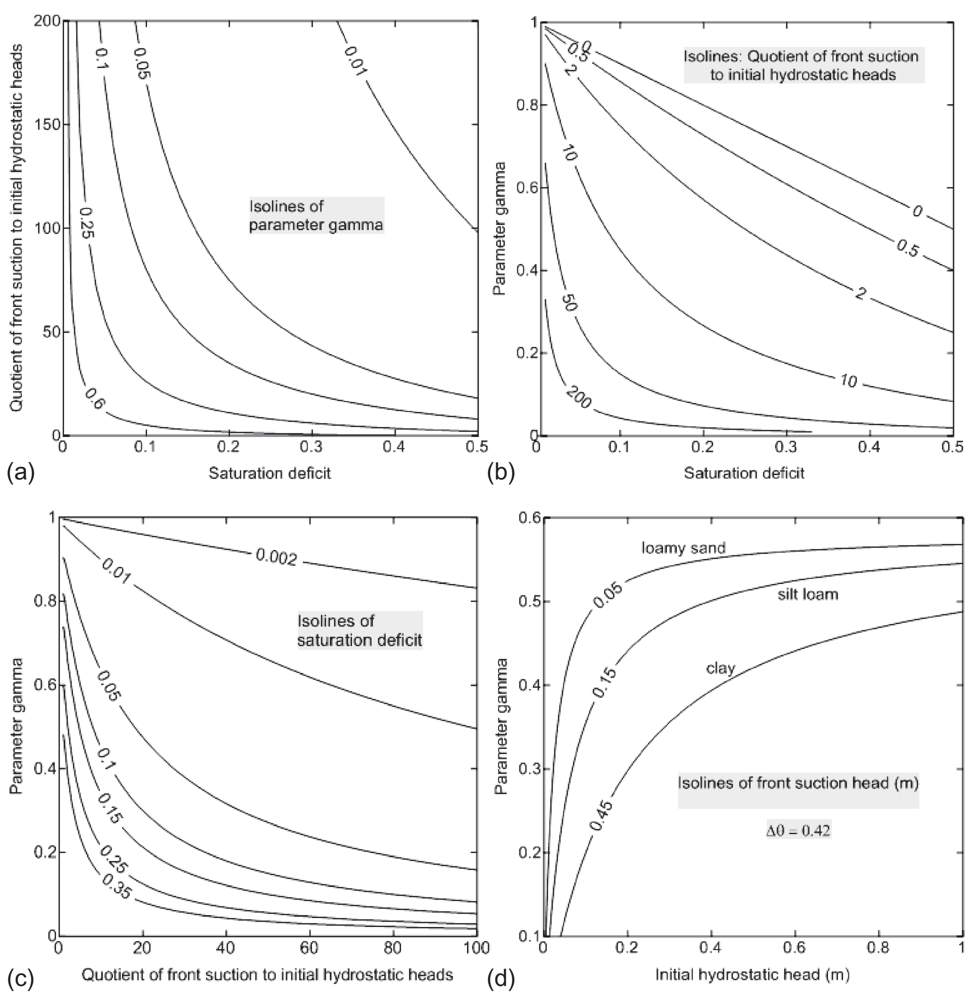

Fig. 3 Functional behaviour of parameter γ.

in γ for rising values of h_0 is more uniform for higher values of suction head. Also, for $h_0 > 0.5$ m, there is a slow variation in γ for all soil types, and a very slow one for medium to coarse soils. We can see that for fixed $\Delta\theta$ and constant ψ along each of the isolines, γ rises asymptotically towards its upper bound $(1 - \Delta\theta)$ when h_0 increases.

We can also put equation (11) in the form:

$$\gamma = \frac{h_0\,(1-\Delta\theta)}{h_0 + \Delta\theta\cdot\psi} = \frac{\alpha h_0}{h_0 + \Delta\theta\cdot\psi} \tag{15}$$

In this equation, the numerator is the product of α (the fraction of soil volume which is unavailable for filling with infiltration water) and h_0 (the total water layer to be infiltrated). Then, the numerator expresses the degree of difficulty for the process to be completed, in terms of the soil layer needed to contain the flooding water layer. In turn, the two terms in the denominator of γ contain the initial conditions which are responsible for speeding up the process: the hydrostatic and suction heads, and the available pore space. The greater either of the terms is, the faster the infiltration will be. Therefore, γ is a dimensionless fundamental number, which accounts for all the initial (extrinsic) conditions affecting the infiltration dynamics. It does not depend on the hydraulic conductivity (intrinsic condition), which acts as an expansion–contraction timing factor regulating the dimensionless time, τ (see equation (10)).

From equation (14), we can see that τ_0 (the dimensionless time needed to complete the infiltration process) depends only on γ and increases along with γ (see Fig. 2). Based on the discussion above, we propose to denote γ as the "infiltration delay parameter". Also, the parameter γ, like a scaled index, ranges between 0 and 1; these bounds correspond, respectively, to initially dry and initially saturated soils.

Let us now express the parameter γ in terms of acting potentials. From equation (5), denoting L_{end} as the final depth reached by the wetting front, we can see that $\Delta\theta$ is equal to (h_0/L_{end}), i.e. the quotient between the initial hydrostatic potential at the surface and the final gravitational potential at the wetting front. By combining equations (5) and (11) we obtain:

$$\gamma = \frac{L_{\text{end}} - h_0}{L_{\text{end}} + \psi} \tag{16}$$

It is easily deduced that this equation contains the total variations of acting potentials—expressed as starting minus initial values—when the flooding water is completely infiltrated. In fact, h_0 is the variation of hydrostatic potential at the soil surface; $L_{\text{end}} = -z_{\text{end}}$ is the variation of gravitational potential at the wetting front, and $-\psi = \psi_f$ expresses the negative variation of suction potential in the wetted soil, which changes from an initially negative value to zero. Denoting these variations as ΔH_p, ΔH_g and ΔH_c, we obtain:

$$\gamma = \frac{\Delta H_g - \Delta H_p}{\Delta H_g - \Delta H_c} \tag{17}$$

where ΔH_g and ΔH_p are positive and ΔH_c is negative.

As may be seen, the parameter γ accounts for the total variations of all acting potentials, relating them in such a way that it expresses the degree of difficulty for a flooding layer to infiltrate, measured in dimensionless space and time scales.

OBTAINING AN EXPLICIT PARAMETRIC SOLUTION OF THE MODEL

Selection of an approximate functional form for $s(\tau)$

Two cases of particular interest in the relationship between s, τ and γ are the limit values $\gamma = 0$ and $\gamma = 1$, for a completely dry soil and saturation, respectively. By taking limits from equation (13), it follows that:

$$s(\tau) \to \begin{cases} 1 - \sqrt{2\tau} & \gamma \to 0 \\ 1 - \tau & \gamma \to 1 \end{cases} \tag{18}$$

In the case of initially saturated soil $\gamma = 1$ and $s(\tau)$ becomes a linear function with slope –1; this, in turn, implies that $h(t)$ becomes a linear function with slope $-K$ which corresponds to the constant infiltration regime at saturation. In this case, the pond empties at $\tau_0 = 1$, while for completely dry soils $(\gamma = 0)$ it is $\tau_0 = 0.5$ (see Fig. 2).

In order to obtain $s(\tau)$ for any intermediate value of γ, a numerical procedure was followed; by assigning to γ incremental values from 0.05 to 0.95, equation (13) was solved for small increments in τ in the interval $[0, \tau_0]$. The Newton-Raphson method was used to solve the equation.

From the table of multiple results, many curves representing $s(\tau)$ for particular values of γ were

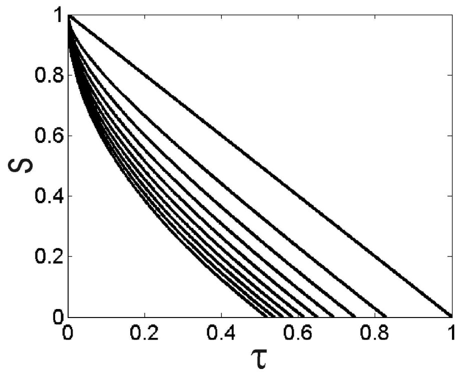

Fig. 4 Relative flooding depth s *vs* dimensionless time, τ, for different values of parameter γ.

derived; these are shown in Fig. 4, where the straight line of unitary slope corresponds to an initially saturated soil for which $\gamma = 1$.

As can be seen from Fig. 4, the factors accompanying the time variable, t, in equation (10) not only allow one to obtain a dimensionless time variable τ, but also act as a grouped reducing factor which bounds the range of τ_0 between 0.5 for completely dry soils and 1 for saturated soils. As expected, higher values of the hydraulic conductivity K and the initial conditions $\Delta\theta$ and ψ shorten the lapse of the infiltration process, while the ponding depth h_0 acts in the opposite way, retarding the completion of that process. We can see from equation (10) that K, $\Delta\theta$ and ψ act as expanding factors of τ, while a greater value of h_0 contracts τ. Therefore, τ also performs as a reduced time variable. However, it is clear from Fig. 4 that the value of parameter γ determines the shape of the curve $s(\tau)$ and, consequently, the infiltration rate and cumulative infiltration at any dimensionless time τ.

A number of candidate parametric functions were adjusted to the curves representing $s(\tau)$. Thereafter, the functions were ranked according to their respective goodness of fit. Among the best, the following function was chosen because it had the lowest number of parameters:

$$s(\tau) = 1 - \left(\frac{\tau}{c}\right)^{b} \tag{19}$$

In order to be consistent with equation (13), in the latter equation the parameters c and b must depend only on γ. In addition, equation (19) must match the functions shown in the right-hand side of equation (18) for both limit cases, i.e. when γ takes the values 0 and 1. These conditions are accomplished when c and b are equal to τ_0, given the relationship between τ_0 and γ shown in Fig. 2. Also, the asymptotic behaviour of τ_0 when γ tends to 1, as shown in Fig. 2, is performed by equation (19) when c and b are equal to τ_0. To ensure the best fit of equation (19) to the analytical implicit solution given by equation (13) for any set of values of initial conditions, it is convenient to make the exponent $b = \tau_0 - a$, where a can be considered as a perturbation of τ_0.

The final obtained function depends implicitly on the parameter γ through τ_0 (see equation (14)) and a:

$$s(\tau) = 1 - \left(\frac{\tau}{\tau_0}\right)^{\tau_0 - a} \tag{20}$$

In order to obtain $a(\gamma)$, several fitting functions were tested. The chosen function was the following:

$$a(\gamma) = \left(\frac{a_1\,\gamma + a_2\,\gamma^2}{1 + a_3\,\gamma + a_4\,\gamma^2}\right) \tag{21}$$

with:

$$\begin{cases} a_1 = 0.05339671 \\ a_2 = -0.05339299 \\ a_3 = -1.32447855 \\ a_4 = 0.34984288 \end{cases} \tag{22}$$

As we can see from the constants $a_1, \ldots, a_4$ found, $a(\gamma)$ in equation (21) takes, respectively, the values 1.467×10^{-5} for $\gamma = 1$ and 0 for $\gamma = 0$, while $\tau_0(\gamma)$ takes correspondingly the values 1 and 0.5 (see equation (14) and Fig. 2). It is clear that a acts always as an almost negligible perturbation of τ_0 in equation (20), just to better fit the relationship given by equation (13).

The function $s(\tau)$ was tested for different values of γ and τ in their respective whole ranges of variation. It was verified that, in all cases, the error in s was less than 7% when comparing equation (20) against the derived function from the original relationship in equation (13). The largest discrepancies occur for the highest values of τ, which demonstrates the quality of the adjustment, keeping in mind that for those values (which mean large times after the beginning of the infiltration) the value of s tends to zero, and for these conditions any adjustment in general is very difficult to control.

Verification of the proposed parameterization

In Fig. 5 we show how this parameterization (equation (20)) fits with the $s(\tau)$ values calculated from the original implicit solution (equation (13)). In this case

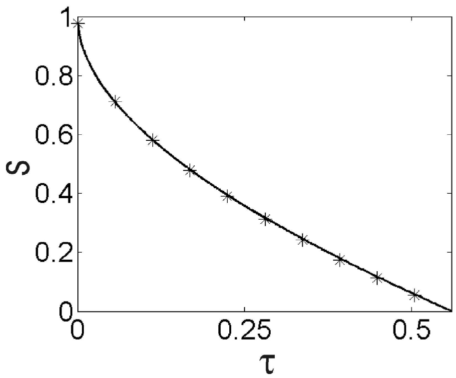

Fig. 5 Performance of the proposed relative flooding depth curve $s(\tau)$. The asterisks show values obtained from Darcy's law (equation (13)). The solid line describes the explicit parametric solution (equation 20). $\gamma = 0.3029$ in this example.

$\gamma = 0.3029$, corresponding to an initially dry loamy sand with $\Delta\theta = 0.4$ and $\psi = 6.13$ cm—according to Rawls *et al.* (1983). We also set $h_0 = 2.5$ cm. As can be seen, the fit is very good. We obtained similar fittings for several other values of γ corresponding to different initial conditions.

Infiltration rates and cumulative storage have also been computed starting from the proposed parameterization for a wide range of different conditions, and compared to the results obtained by an iterative procedure for solving equation (6). To do this, five different cases were chosen by combining different values of ψ, $\Delta\theta$, K and h_0 in order to get values of γ covering its whole range of variation (between 0 and 1). These cases are shown in Table 1; the two first cases correspond to loamy sand, the third one to silt loam and the last two to clay. The data of suction head and effective porosity were taken from Rawls *et al.* (1983), and the values of $\Delta\theta$ were chosen as the effective porosity and half of this value for each type of soil texture.

We also show the inverse values of the time scaling factor $(K\chi/h_0)$. As may be observed, these values vary in several orders of magnitude, from seconds to weeks. The range of variation could be even greater if values of the hydraulic conductivity are taken at the extremes of the experimental range. Rawls and Brakensiek (1989) suggested that the hydraulic conductivity at saturation can vary from 3×10^{-9} m/s for certain clays to more than 3×10^{-5} m/s for coarse sands.

As shown in Table 2, the parameterization is very satisfactory for such diverse soil types and initial conditions. As mentioned before, the largest relative discrepancies occur for long periods of time after the beginning of the infiltration, which implies that the remaining ponding depth is already very low.

All the above calculations allow one to determine $h(t)$ for any conditions, but assuming that there is no saturated soil layer immediately below the surface. If this was the case, it would be necessary to calculate first h_0 corresponding to the moment when infiltrations starts; to do this, the amount of water already infiltrated must be calculated using equation (5), then added to the ponding depth. Then, χ and γ can be computed using equations (8) and (11). Finally, s and τ should be calculated by means of equation (13) using the new initial time.

In order to reduce the quantity of required parameters, the suction head at the sharp wetting front can be estimated from the initial soil water content value by applying empirically fitted relationships such as the ones of Brooks and Corey (1964), Van Genuchten (1980) or Morel-Seytoux *et al.* (1999), who proposed two modifications to the Brooks-Corey curve, one at each end of the suction range.

CONCLUSIONS

The theoretical formulation of the infiltration process for the Green-Ampt scheme has been extended to decreasing flooding depth in an isolated system, starting from Darcy's equation applied to the "sharp wetting front" scheme. By defining dimensionless variables of flooding depth, s, and time, τ, the model was reduced to a simple-infiltration-rate dimensionless differential equation with one parameter involved, γ, which accounts for the initial ponding depth, saturation deficit and suction head. The implicit solution $\tau(s)$ was obtained through integration.

A dimensionless time τ was proposed, where the factor accompanying time t includes the hydraulic conductivity K and the initial conditions h_0, $\Delta\theta$ and ψ. In consistency with the physical meaning of these terms, they are arranged in the proposed expression for τ in such a way that K, $\Delta\theta$ and ψ act as expanding factors of τ, while a greater value of h_0 contracts τ. It was also shown that τ performs as a reduced time variable.

It was proven that τ_0, the dimensionless time needed to complete the infiltration process, depends only on γ, and increases with γ.

Concerning the parameter γ, its characteristics and functional behaviour were analysed, and its physical meaning was discussed. It was shown that γ which is a function of the initial conditions h_0, $\Delta\theta$ and ψ, is a scaled index ranging from 0 to 1 for, respectively, initially dry and initially saturated soils. It was proven that the value of parameter γ determines the shape of the curve $s(\tau)$, and,

Table 1 Values of γ and the inverse of the time scaling factor $h_0/(K\chi)$ calculated for different soil types and initial conditions

Case	Initial depth h_0 (10^{-2}m)	Saturation deficit, $\Delta\theta$	Suction head, ψ (10^{-2}m)	Hydraulic conductivity, K (10^{-6} m/s)	γ	$h_{0}/(K\chi)$ (10^4 s)
1	0.1	0.401	6.13	8.31	0.0234	0.00047
2	10.0	0.201	6.13	8.31	0.7120	1.07140
3	10.0	0.486	16.68	1.81	0.2839	3.05130
4	0.1	0.423	29.22	0.14	0.0046	0.05733
5	10.0	0.212	29.22	0.14	0.4866	44.10600

Table 2 Comparison of values of remaining flooding depths and infiltration rate for different cases, according to Darcy (equation (6)) and starting from the obtained parametric function (equation (20))

Case	Time (10^4 s)	h (eq. 6) (10^{-3} m)	h (eq. 20) (10^{-3} m)	Error (%)	$\mathrm{d}h/\mathrm{d}t$ (eq. 6) (10^{-7} m/s)	$\mathrm{d}h/\mathrm{d}t$ (eq. 20) (10^{-7} m/s)	Error (%)
1	0.00036	0.6855	0.6857	0.04	−6652	−6661	0.13
1	0.00181	0.2945	0.2942	0.11	−2990	−2992	0.07
1	0.00333	0.0517	0.0516	0.22	−2238	−2233	0.19
2	0.10714	76.7050	77.6283	1.47	−187	−185	4.30
2	0.53568	37.3027	36.2853	2.73	−109	−111	2.11
2	0.96423	7.0433	6.6222	5.98	−95	−90	5.13
3	0.30513	70.7336	71.0799	0.49	−90	−92	1.64
3	1.52570	31.5683	31.1657	1.28	−43	−44	0.85
3	2.74620	5.6707	5.5187	2.68	−34	−33	2.26
4	0.00573	0.6841	0.6842	0.01	−550	−550	0.03
4	0.02866	0.2932	0.2932	0.02	−246	−246	0.01
4	0.05159	0.0514	0.0514	0.04	−184	184	−0.04
5	4.41060	72.9660	73.6402	0.93	−6	−6	2.88
5	22.0530	33.7678	33.0599	2.10	−3	−3	1.47
5	39.6960	6.1955	5.9186	4.47	−2	−2	3.80

consequently, the infiltration rate and cumulative infiltration at any dimensionless time τ.

We concluded that parameter γ, which we propose to call the "infiltration delay parameter", is a fundamental number which expresses the degree of difficulty for the infiltration process to be completed, measured in dimensionless space and time scales. Also, it was shown that the parameter γ can be expressed in a simple way as a function of the total variations of all acting potentials: gravitational, hydrostatic and capillary.

The implicit solution obtained implies a significant advancement with respect to the equation derived by Philip (1992), which is in dimensional form and depends on four parameters: the three initial conditions included in γ and the hydraulic conductivity included in τ. Additionally, the implicit infiltration depth equation obtained allows the elapsed dimensionless time τ_0 to be derived when the pond empties; in this expression τ_0 depends only on the parameter γ, showing the functionality relating both variables. Philip (1992) derived an equivalent, but more complex, equation in which the time corresponding to τ_0 depends on the four parameters included in γ and τ.

Additionally, an explicit parametric solution was proposed to approximate the implicit solution and was shown to perform well for a wide range of parameter values covering the conditions present in nature. The equation obtained for $s(\tau)$ depends only on the descriptive parameter γ.

Acknowledgements The work upon which this article is based was partially supported by funds from CONICET, CONAE, Universidad Tecnológica Nacional (PID 25D027), Universidad de Buenos Aires (UBACYT G813) and Universidad Nacional del Litoral (PI 25-140). We thank Ghislain de Marsily from Université Pierre et Marie Curie, France, for their valuable suggestions and corrections to the original manuscript.

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